Stress Test

RISK Management and Financial Steering

Reevaluation of the capital charge in insurance after a large shock: empirical and theoretical views

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Joint work with F. Borel-Mathurin (ACPR/EIOPA) & J. Segers (Louvain)

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Enterprise Risk Management (ERM): a revised definition

- COSO: Enterprise Risk Management (ERM) is a process, effected by an entity's board of directors, management and other personnel, applied in strategy-setting and across the enterprise, designed to identify potential events that may affect the entity, and manage risk to be within its risk appetite, to provide reasonable assurance regarding the achievement of entity objectives.

- COSO 2016: ERM corresponds to the culture, capabilities and practices integrated with strategy-setting and its execution, that organizations rely on to manage risk in creating, preserving and realizing value.

- Importance of Risk Culture!
- Risk appetite: the level of risk that an insurer is ready to take in order to achieve strategic objectives

SCOR 2009 Risk appetite (Institut des Actuaires)

SCOR’s risk tolerance is derived from its risk appetite

Solvency

- SCOR’s **risk measure for solvency is 99%Tail Value at Risk** (TVaR), corresponding to a financial security level in line with the target rating of A+ (*S&P*) and A (*A.M. Best*) (corresponding to a ruin probability of 1:250)

Diversification

- **No risks (LOB, Asset Class) must consume more than 5% of available capital** when looking at the 95%TVaR
- **No extreme scenario** (with a probability of higher or equal to 1:250) **must result in a loss larger than 15% of available capital**

Compliance

➔ **Full compliance** with all regulatory and solvency requirements (US RBC, Swiss Solvency Test, EU Solvency II etc.)

ORSA & Risk appetite in insurance

- Own Risk & Solvency Assessment
- Pillar 2 of Solvency II
- Also exists in SST, Australia & North America
- MUST contain stress tests (defined by supervisor AND defined by entity)

Different types of scenarios

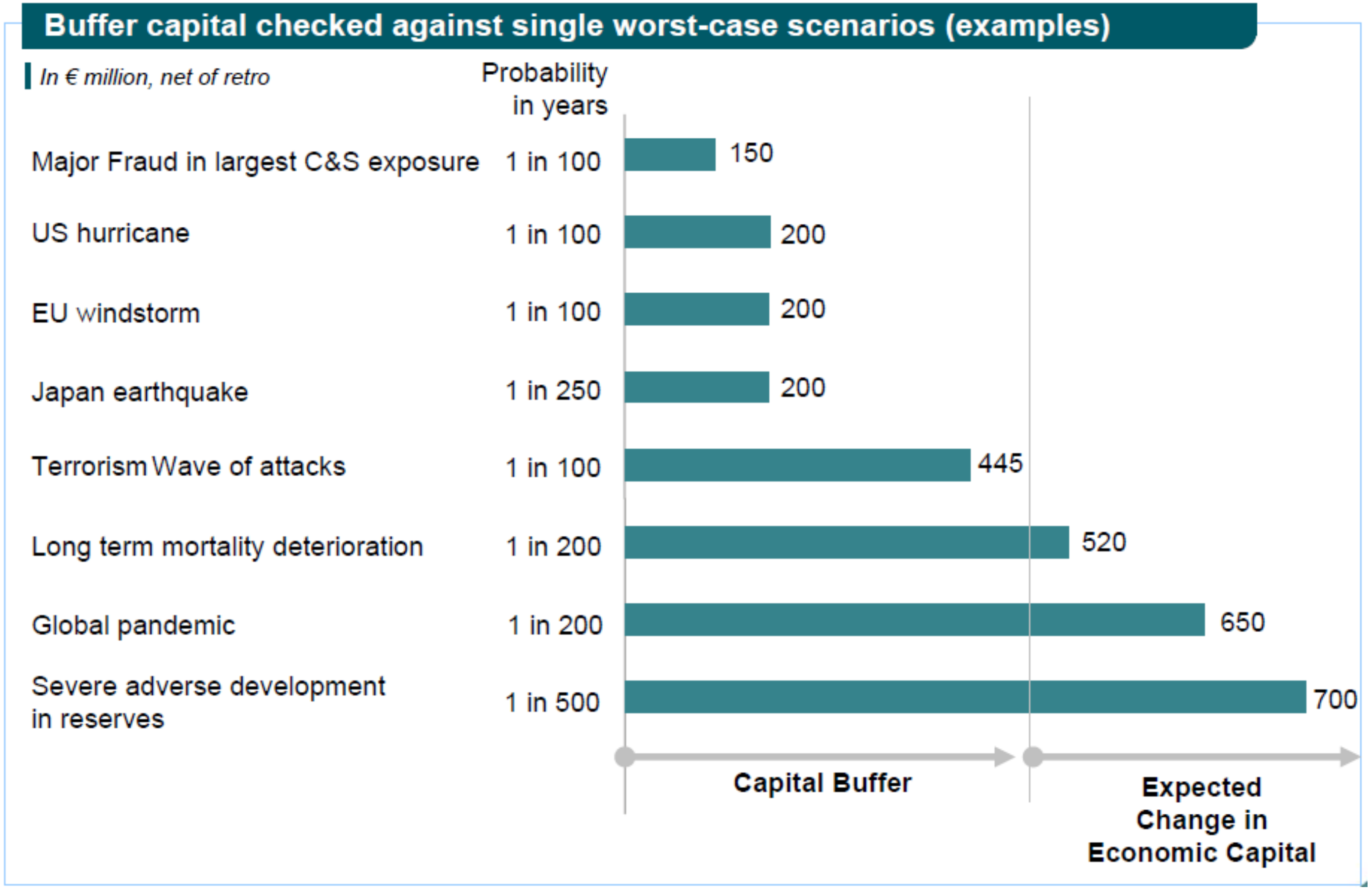
- From supervisors:
  - for individual entity risk assessment
  - for Solvency requirements
  - for financial stability/systemic risk assessment
- From companies top management and board members
  - ORSA
  - Reverse-stress tests
  - Worst-case scenarios

Ways to calibrate scenarios

- External reference
- Statistical / econometric approach when available
    - Use of MSCI World Index annual returns (1973-2009), non-parametric

    Reference: Equity risk sub-module (former Consultation Paper 69) 29 January 2010, Article 111 and 304 -Level2 Implementing Measures
- Model-free, really?
- Panel of experts
- Lamfalussy approach / QIS
- Reference to previous events: 1918 Flu Epidemics
- Cat- or pandemic model
- Reverse-stress tests: what would put the company in the red?
- Worst-case scenarios: what is the maximal exposure?

Additional stress tests and what-if scenarios at SCOR (2009)



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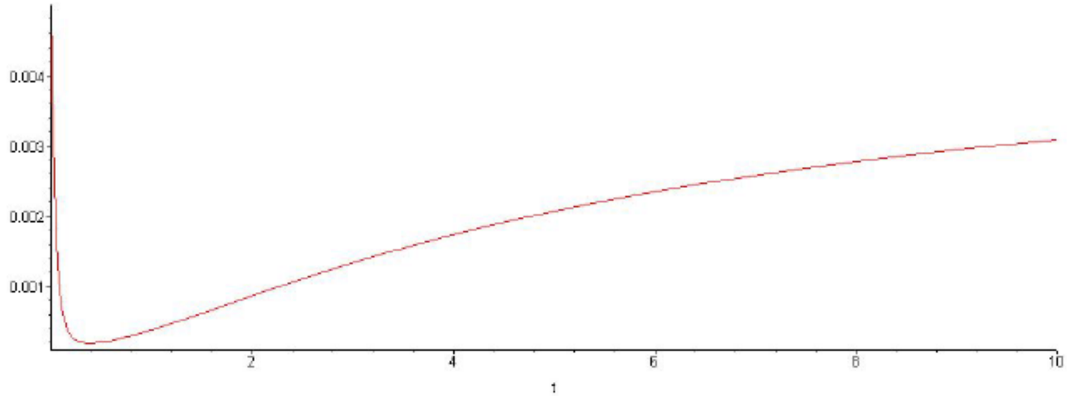
Pitfalls

- Obvious difficulty to measure likelihood level
- Definition of Worst-case scenario
- What-if scenarios:
  - Tail probability re-assessment
  - Loss-Absorbing Capacity re-assessment
  - Decisions of top management
- Limits of conditioning with respect to events like {X=x}

Co-Value-at-Risk



β such that $CoVaR_\alpha(Y \mid X) = VaR_\beta(Y)$ as a function of correlation parameter θ for the Clayton copula. The lower β, the higher the required capital.

Some first observation in a joint paper with Alexandre Mornet

Storms in France

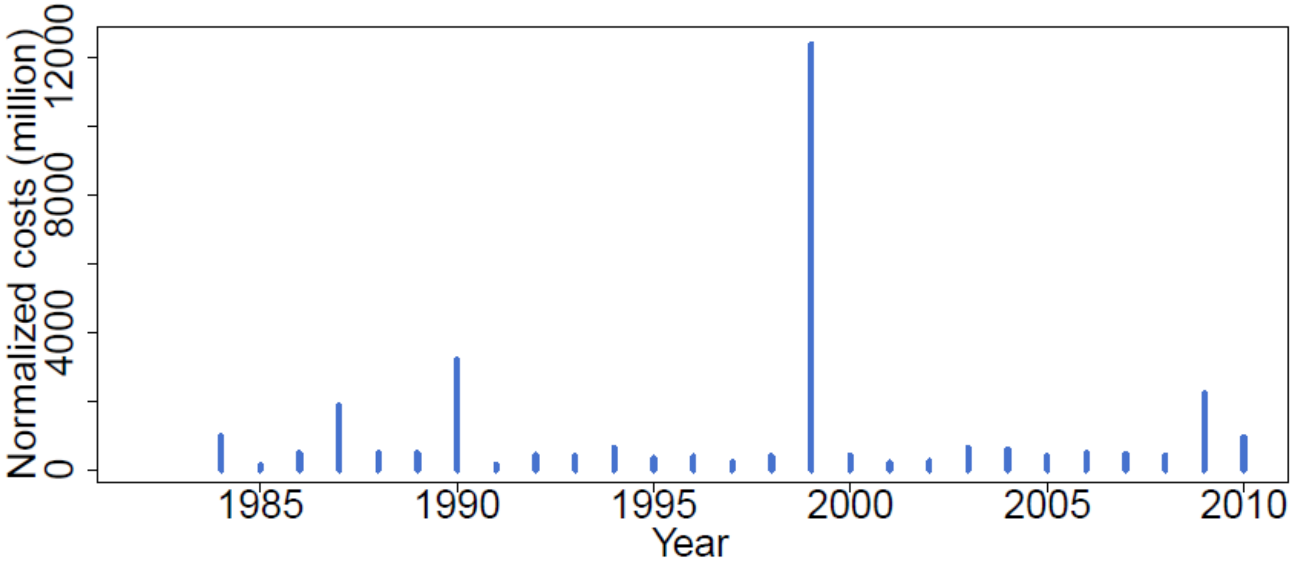


Figure 1 Updated annual storm costs in France since 1984

Return periods: with and without Lothar

Table 2 Calculations of return period according to different scenarios

Period	1970-2013				1993-2013
Threshold : u	10	20	10wL	10wM	10
Lothar Return Period	93	88	280	160	31
L(1.5) Return Period	136	128	490	252	84

ACPR / EIOPA Stress tests 2014

Quantile Re-assessment is NOT compulsory after the stress test!

Not really a what-if exercise

Some stakeholders were persuaded that not reassessing the Capital Requirement was precautionary

What can we say from a theoretical perspective, and in practice from stress test results?

Insurer simplified balance-sheet in Solvency II

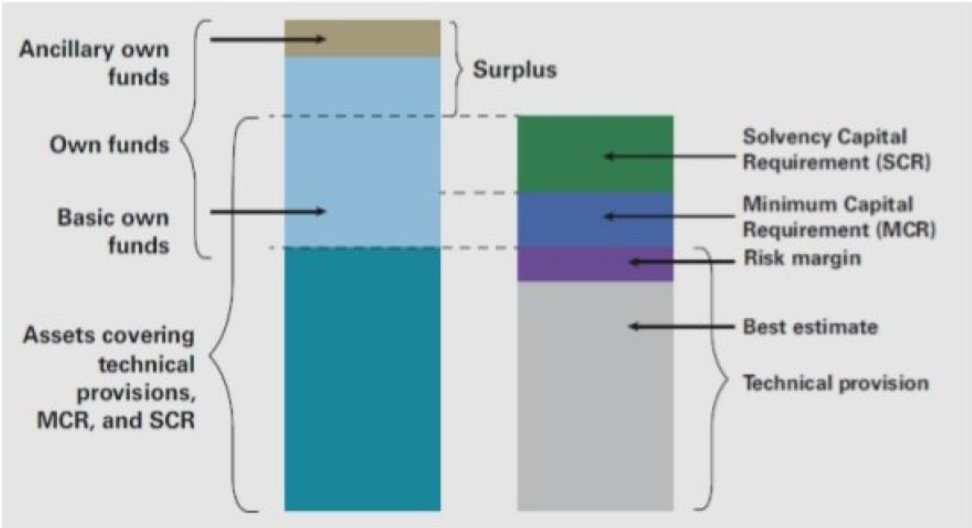


Figure 1: Insurer simplified Balance Sheet (Solvency II) (source: UK actuaries)

Risk taxonomy in Solvency II

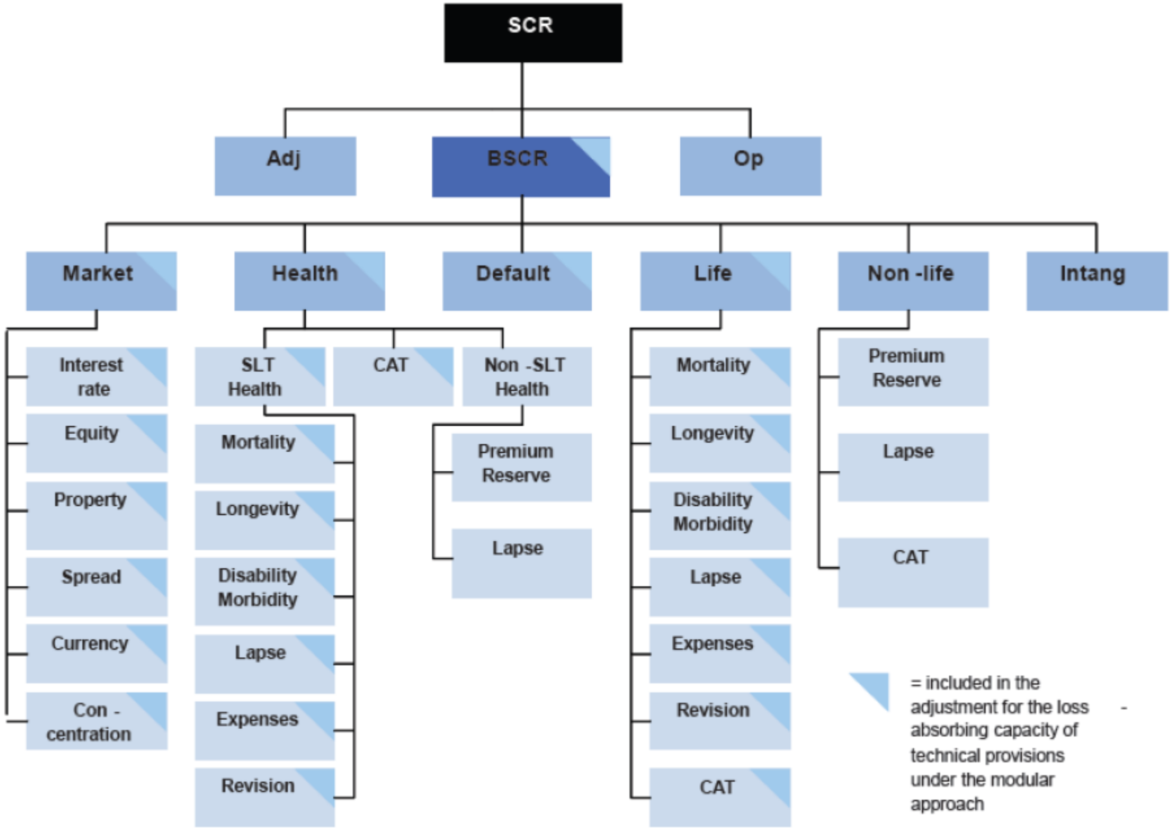


Figure 2: SCR: risk modules breakdown (Source: EIOPA)

Stressed balance sheet

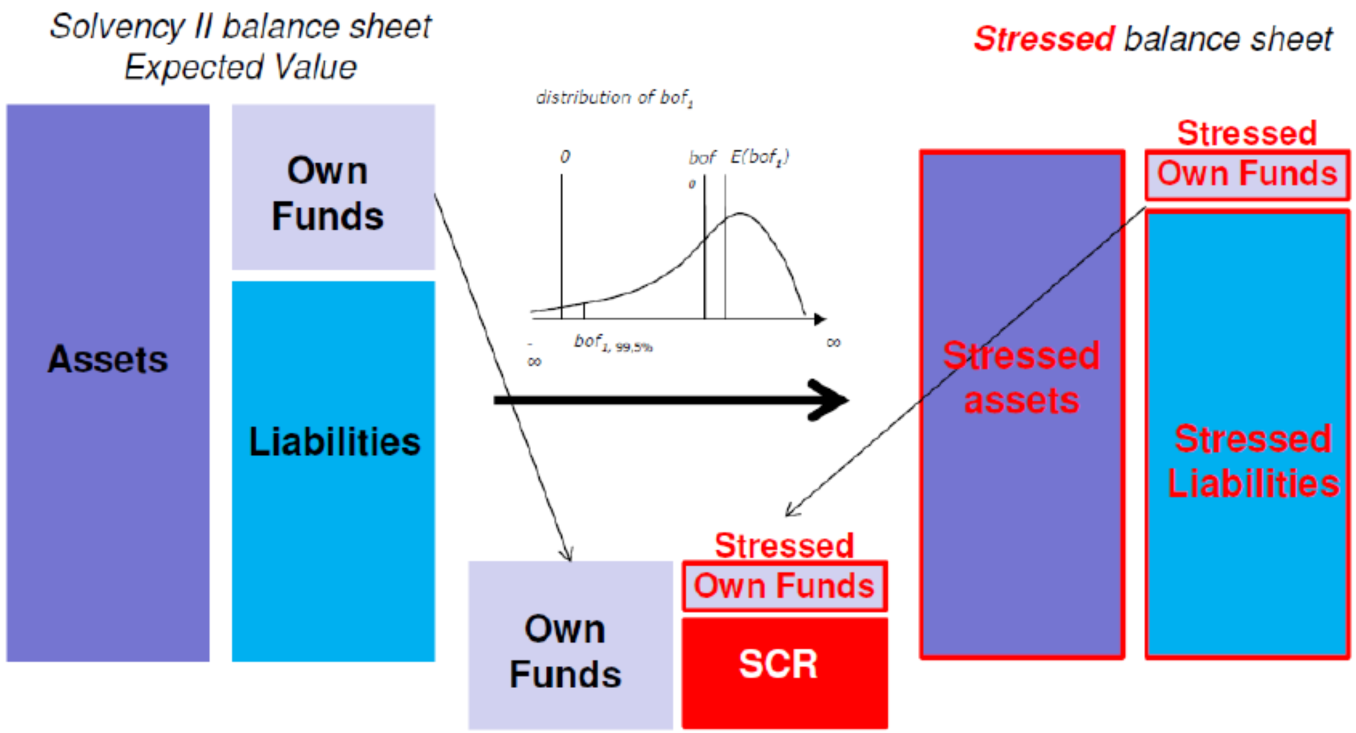


Figure 3: SCR risk sub-modules calculation (Source: ACPR)

Technical provisions breakdown in Europe

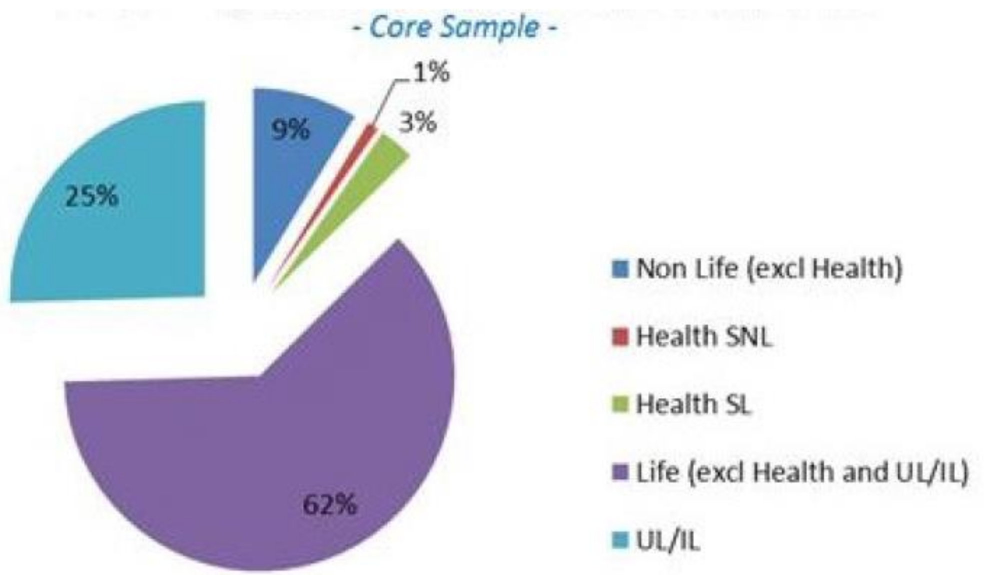


Figure 4: Technical provisions breakdown (source: EIOPA Stress Test 2014)

Decomposition of pre-stress SCR

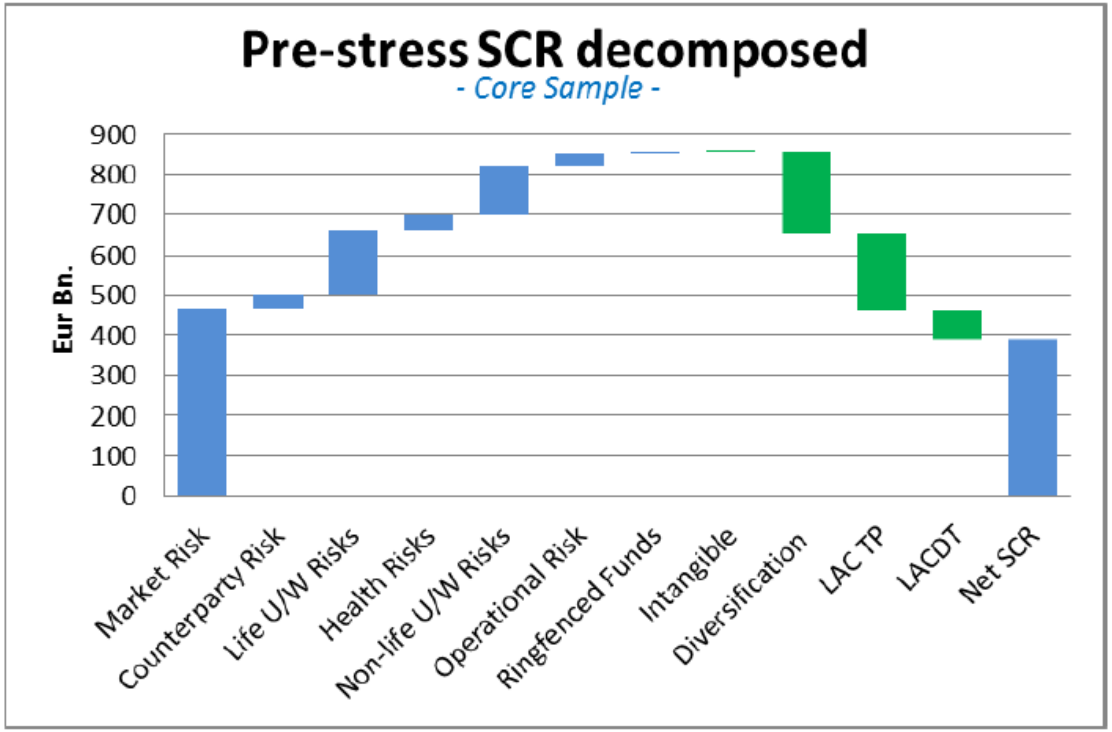


Figure 5: SCR Decomposition (source: EIOPA Stress Test 2014)

However, this one-to-one correspondence is not actually observed in the 2014 Stress test data (European Insurance and Occupational Pensions Authority, 2014): although very few undertakings reassessed their SCR post-stress – less than 30%, the reassessment was optional – a significant share (more than 40%) of the undertakings underwent an increase of their global net SCR in at least one of the market scenarios.

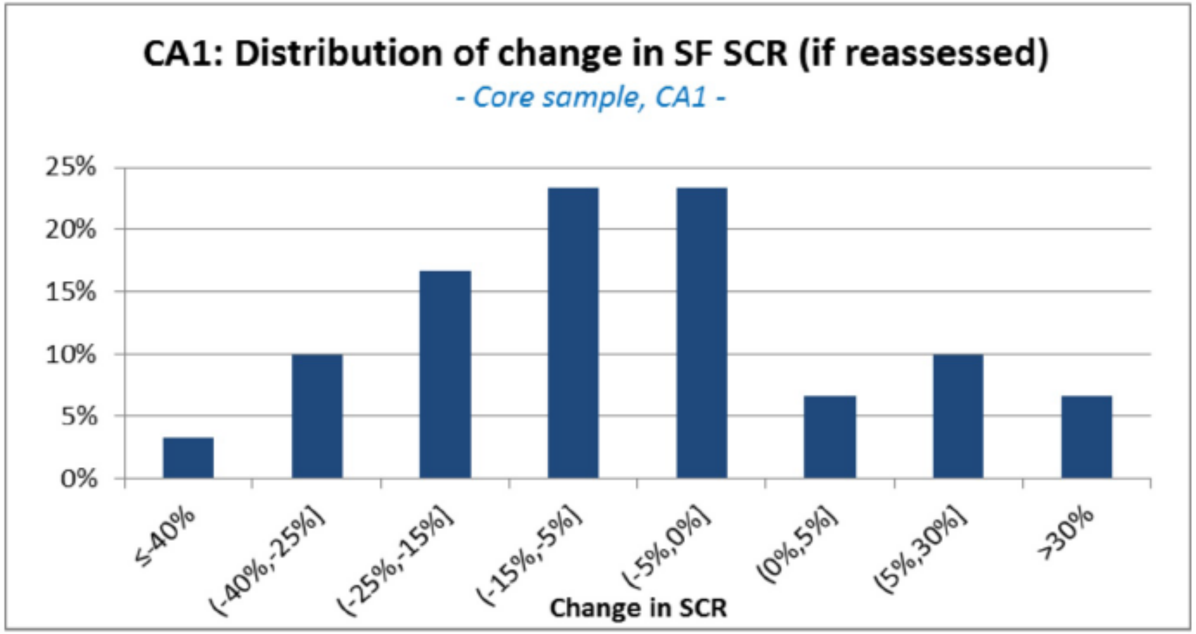


Figure 6: Distribution of reassessed SCR (source: EIOPA Stress Test 2014)

A simplified model

In this simplified model, we consider that the SCR is given by

$$\mathrm{SCR} = [\mathrm{VaR}_{99.5\%}(X) - \mathrm{E}(X) - b]_+ , \tag{3.1}$$

where X is a random variable corresponding to the 1-year random loss the insurer may face. Here, for simplification purposes, we consider only one risk factor, which can be financial or P&C cat. Of course, in the real world, there are many risk factors, aggregated either with the standard formula or by means of an internal model. We shall discuss the impact of diversification on our results in the sequel. The parameter b plays an important role: it corresponds to the loss-absorbing capacity, and it is likely to be affected if a large event occurs.

After the stress test

$$\text{SCR} = [\text{VaR}_{99.5\%}(X) - \text{E}(X) - b]_+$$

After a shock, b is transformed into b' and X is transformed into

$$X' = a\tilde{X}, \tag{3.2}$$

where a is a factor accounting for the change in the exposure, and $\tilde{X}$ is the revised version of X after taking the last shock into account.

Theoretical insight from EVT

We take a P&C view on the random loss X underlying the SCR calibration. Let $X_1, X_2, \ldots$ be i.i.d. random variables corresponding to observations of X. For simplicity, assume that their common distribution is continuous. Denote the ascending order statistics of $X_1, \ldots, X_n$ by $X_{n:1} < \ldots < X_{n:n}$.

Consider statistics of the type

$$T_n = t_n(X_1, \ldots, X_n),$$

where $t_n : \mathbb{R}^n \to \mathbb{R}$ is a permutation invariant function. Think of T_n as an estimator of some tail-related quantity: a tail quantile, a return level, ... The statistic T_n depends on the data only through the order statistics:

$$T_n = t_n(X_{n:1}, \ldots, X_{n:n}).$$

Estimation after a record: no distorsion!

First, assume that the record occurs at "time" n, that is, $X_n > X_{n-1:n-1}$, or, in other words, the rank of X_n among $X_1, \ldots, X_n$ is equal to n. At a given sample size, the vector of order statistics is independent of the vector of ranks. We find that

$$[T_n \mid X_n > X_{n-1:n-1}] \stackrel{d}{=} T_n. \tag{4.1}$$

That is, computing the statistic right *after* a record does not lead to any distortion.

Estimation excluding the record: distorsion!

Second, assume that we compute the statistic right *before* a record occurs. Specifically, suppose that X_{n+1} is a record: $X_{n+1} > X_{n:n}$. How does the occurrence of that event affect the distribution of T_n?

If X_{n+1} is a record in the stretch $X_1, \ldots, X_{n+1}$, then $X_i < X_{n+1}$ for all $i = 1, \ldots, n$, and the vector of order statistics $(X_{n:1}, \ldots, X_{n:n})$ is equal to the vector $(X_{n+1:1}, \ldots, X_{n+1:n})$. It follows that

$$[(X_{n:1}, \ldots, X_{n:n}) \mid X_{n+1} > X_{n:n}] \stackrel{d}{=} (X_{n+1:1}, \ldots, X_{n+1:n}). \tag{4.2}$$

Equation (4.2) implies that

$$[T_n \mid X_{n+1} > X_{n:n}] \stackrel{d}{=} t_n(X_{n+1:1}, \ldots, X_{n+1:n}). \tag{4.3}$$

Computing the statistic right *before* the occurrence of a record has a clear impact on its distribution: compare (4.1) and (4.3).

$$[T_n \mid X_n > X_{n-1:n-1}] \stackrel{d}{=} T_n. \tag{4.1}$$

Example 1 (Tail probability)**.** Let u be a high level. Aim is to estimate the tail probability $p = 1 - F(u)$. Note that the return level is equal to $1/p$. The simplest possible estimator is the empirical one,

$$T_n = \frac{1}{n} \sum_{i=1}^{n} I(X_i > u).$$

Clearly, the estimator is unbiased:

$$\mathrm{E}[T_n] = p.$$

However, if we ignore the information that at time $n+1$, a new record occurred, then

$$\begin{aligned}
\mathrm{E}[T_n \mid X_{n:n} < X_{n+1}] &= \mathrm{E}\left[\frac{1}{n} \sum_{i=1}^{n} I(X_{n+1:i} > u)\right] \\
&= \mathrm{E}\left[\frac{1}{n} \sum_{i=1}^{n+1} I(X_{n+1:i} > u) - \frac{1}{n} I(X_{n+1:n+1} > u)\right] \\
&= \frac{n+1}{n} p - \frac{1}{n}\{1 - (1-p)^{n+1}\}.
\end{aligned}$$

The expected relative error is therefore

$$\frac{1}{p} \mathrm{E}[T_n \mid X_{n:n} < X_{n+1}] - 1 = \frac{1}{n} - \frac{1 - (1-p)^{n+1}}{np}.$$

If $u = u_n \to \infty$ in such a way that $np = np_n = n\{1 - F(u_n)\} \to \tau \in (0, \infty)$, i.e., if $p \sim \tau/n$, then the expected relative error converges to a nonzero limit:

$$\frac{1}{p} \mathrm{E}[T_n \mid X_{n:n} < X_{n+1}] - 1 \to -\frac{1 - e^{-\tau}}{\tau}, \qquad n \to \infty. \tag{4.4}$$

Example 2 (Tail-quantile estimator)**.** Let Q be the quantile function of F. The aim is to estimate a tail quantile, $Q(1-p)$, where the tail probability, $p \in (0,1)$, is small. Assume that F is in the domain of attraction of the Fréchet distribution with shape parameter $\alpha \in (0,\infty)$. We will only use classical tools of extreme value theory. The interested reader may consult for example the book of Beirlant *et al.* (2006) for a presentation of the Fréchet domain of attraction. Let $\gamma = 1/\alpha$ be the extreme-value index. Let $k \in \{1,\ldots,n-1\}$ be such that $p < k/n$. A common estimator is based on the approximation

$$Q(1-p) \approx Q(1-k/n)\,\{(k/n)/p\}^{\gamma}.$$

On a logarithmic scale, the estimator takes the form

$$\log \widehat{Q}_{n,k}(1-p) = \log X_{n:n-k} + \widehat{\gamma}_{n,k} \log\{(k/n)/p\}, \tag{4.5}$$

where $\widehat{\gamma}_{n,k}$ is an estimator of the extreme-value index γ, for instance the Hill estimator

$$\widehat{\gamma}_{n,k} = \frac{1}{k}\sum_{i=1}^{k} \log X_{n:n-i+1} - \log X_{n:n-k}. \tag{4.6}$$

(We implicitly assume that $X_{n:n-k} > 0$.)

Combining (4.5) and (4.6), we find that the tail quantile estimator is linear in the order statistics $Y_{n:n-k} < \ldots < Y_{n:n}$, where $Y_i = \log X_i$. Identity (4.3) then permits in principle to calculate its conditional distribution on the event that $X_{n:n} < X_{n+1}$:

$$\begin{aligned}
&[\log \widehat{Q}_{n,k}(1-p) \mid X_{n:n} < X_{n+1}] \\
&\qquad \stackrel{d}{=} \log X_{n+1:n-k} + \left(\frac{1}{k}\sum_{i=1}^{k} \log X_{n+1:n-i+1} - \log X_{n+1:n-k}\right) \times \log\{(k/n)/p\}.
\end{aligned}$$

To evaluate the impact of ignoring a known record, let us compute the expectation of the estimator under the simplifying assumption that the random variables X_i are iid Pareto with shape parameter α, that is, $F(x) = 1 - x^{-\alpha}$ for $x \geq 1$. Equivalently, the random variables Y_i are iid Exponential with expectation equal to γ. In that case, $\log Q(1-p) = \gamma \log(1/p)$. A well-known representation of the order statistics from an exponential distribution yields

$$\mathrm{E}[Y_{n:n-j+1}] = \gamma \left(\frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{j} \right), \qquad j \in \{1, \ldots, n\}. \tag{4.7}$$

Equation (4.7) yields the following expressions for the expectation of the estimator of the log tail quantile. Unconditionally, we have

$$\mathrm{E}[\log \widehat{Q}_{n,k}(1-p)] = \log Q(1-p) + \gamma \left(\frac{1}{n} + \cdots + \frac{1}{k} - \log(n/k) \right).$$

$$\mathrm{E}[\log \widehat{Q}_{n,k}(1-p)] = \log Q(1-p) + \gamma \left(\frac{1}{n} + \cdots + \frac{1}{k} - \log(n/k) \right).$$

The second term on the right-hand side converges to zero relatively quickly. In contrast, conditionally on the occurrence of a record on the next day, we have

$$\begin{aligned} &\mathrm{E}[\log \widehat{Q}_{n,k}(1-p) \mid X_{n:n} < X_{n+1}] \\ &\qquad = (1-a_k) \log Q(1-p) + \gamma \left(\frac{1}{n} + \cdots + \frac{1}{k} - (1-a_k)\, \log(n/k) \right), \end{aligned}$$

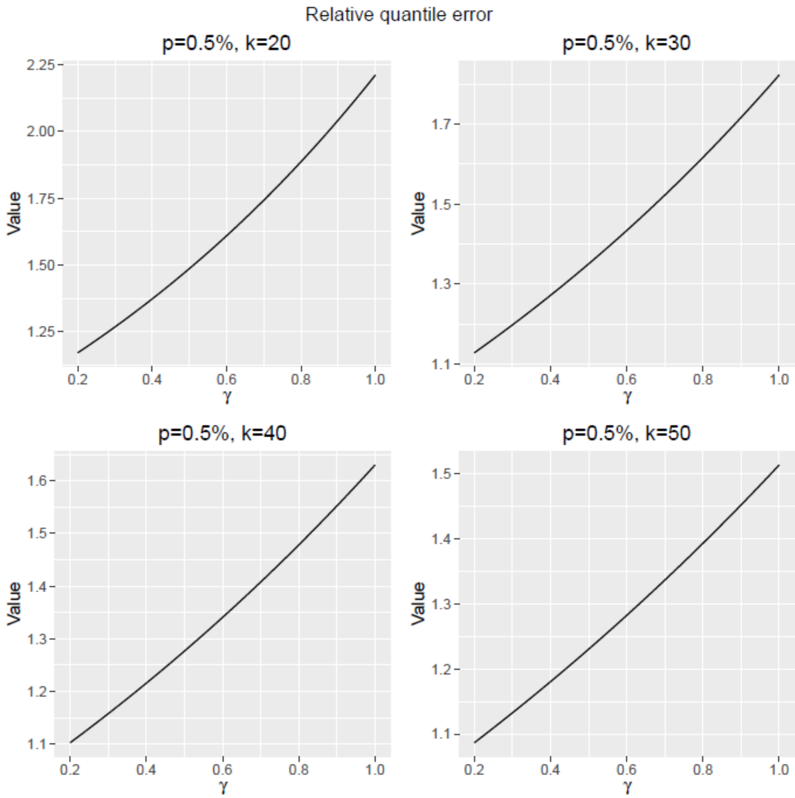
where

$$a_k = \frac{1}{k} \sum_{j=1}^{k} \frac{1}{j+1}.$$

The sequence a_k tends to zero as k tends to infinity: $a_k \sim \log(k)/k$ as $k \to \infty$. Still, since the relative error occurs on the logarithmic scale, there is potentially a severe underestimation of the tail quantile: indeed, we have $(1-a_k) \log Q(1-p) = \log[\{Q(1-p)\}^{1-a_k}]$.

The relative error is thus given by $\{Q(1-p)\}^{-a_k} = (1/p)^{a_k \gamma}$. The larger the tail index γ and the smaller the tail probability p, the larger the relative error. The result remains valid for the more general Pareto distribution $F(x) = 1 - (x/\sigma)^{-\alpha}$ for $x \geq \sigma$, where $\sigma > 0$ is a scale parameter.

Illustrations on real-world examples



The case a<1 (stock crash, ...)

Liabilities	100	$M€$
gBSCR	7.5	$M€$
b	5.25	$M€$
Net SCR	2.23	$M€$

Table 1: Toy company, pre-stress situation (source: ST 2014 figures)

in $M€$	ST $(a \approx 0.93)$	$a = 0.9$	$a = 0.8$
Liabilities'	97.5	96.8	86
BSCR'	7.17	6.7	6
b'	4.45	4.02	3.27
Net SCR'	2.71	2.71	2.71

Table 2: Toy company, post-stress situation (source: ST 2014 figures, authors' calculations)

The case a>1 (earthquake, flooding, mass non-lapse,…)

To illustrate this point, we choose for b a market average and $a = 1.2$. So far, this figure has been provided as a percentage of the aggregate basic solvency capital requirement both for the participants of the 2014 EIOPA ST (European Insurance and Occupational Pensions Authority, 2014) and their French counterparts (Borel-Mathurin and Gandolphe, 2015). The absorption capacity is $b = 38\% \times \overline{\text{gBSCR}}$ (resp. $b = 61\% \times \overline{\text{gBSCR}}$) for the whole setup of european groups participants (resp. the French groups). For values of gross BSCR ranging from 50% to 150% of the market average gross SCR, we plot in Figure 10 the sub-regions of the half-plane (b', gross BSCR) where the re-evaluated SCR is larger than the initial one.

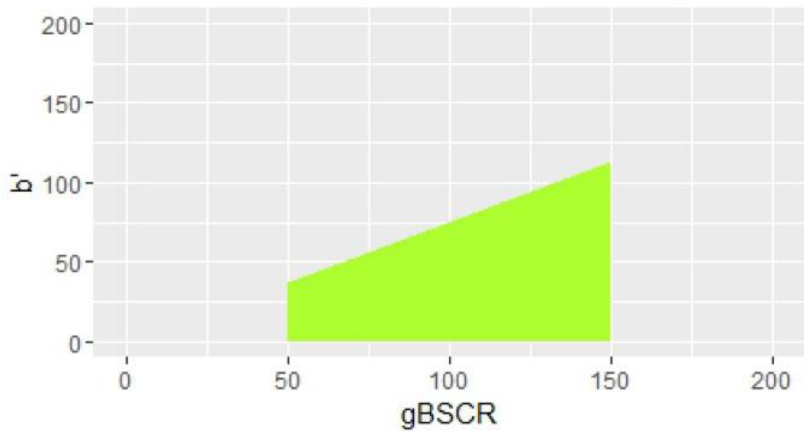


Figure 10: b' value with a positive increase of the net SCR

Potential scissors effect on SCR coverage ratio

$$\mathrm{RM}' = \mathrm{CoC} \sum_{t \geq 0} \mathrm{SCR}'(t).$$

To determine $\mathrm{SCR}(t)$, one may either project at each timestep t and make a complete Best Estimate (BE) determination in future time, or use one of the simplified methodologies, for example the proportional approach based on the Best Estimate[13]. We can then infer the future value of the SCR:

$$\mathrm{SCR}(t) = \mathrm{SCR}(0) \times \frac{\mathrm{BE}(t)}{\mathrm{BE}(0)} \quad \text{and} \quad \mathrm{RM} = \mathrm{CoC} \times \mathrm{SCR}(0) \sum_{t \geq 0} \frac{\mathrm{BE}(t)}{\mathrm{BE}(0)}.$$

Assume that the proportions stay constant after stress, so that

$$\forall t \geq 0, \quad \frac{\mathrm{BE}'(t)}{\mathrm{BE}'(0)} = \frac{\mathrm{BE}(t)}{\mathrm{BE}(0)}.$$

Then

$$\mathrm{RM}' = \mathrm{CoC} \times \mathrm{SCR}'(0) \sum_{t>0} \frac{\mathrm{BE}'(t)}{\mathrm{BE}'(0)} = \frac{\mathrm{SCR}'(0)}{\mathrm{SCR}(0)} \mathrm{RM}\,.$$

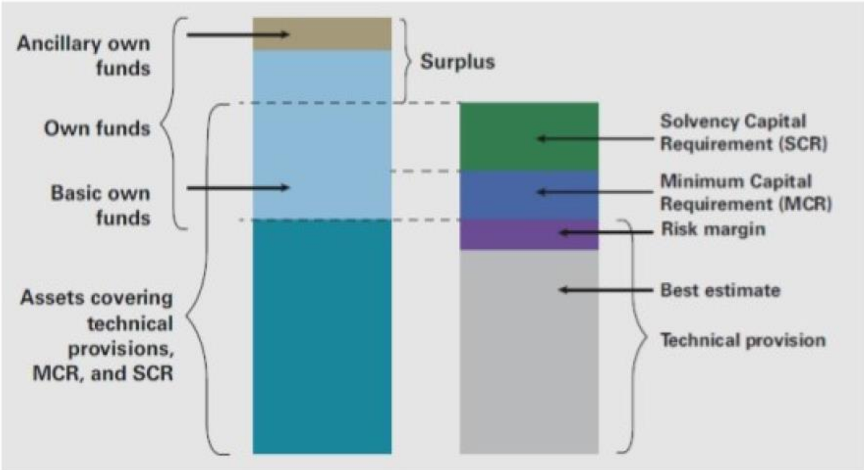


Figure 1: Insurer simplified Balance Sheet (Solvency II) (source: UK actuaries)

Conclusion / perspectives

- Policy implications

- Still work in progress

- Papers

- Next steps: diversification, re-estimation and LAC's

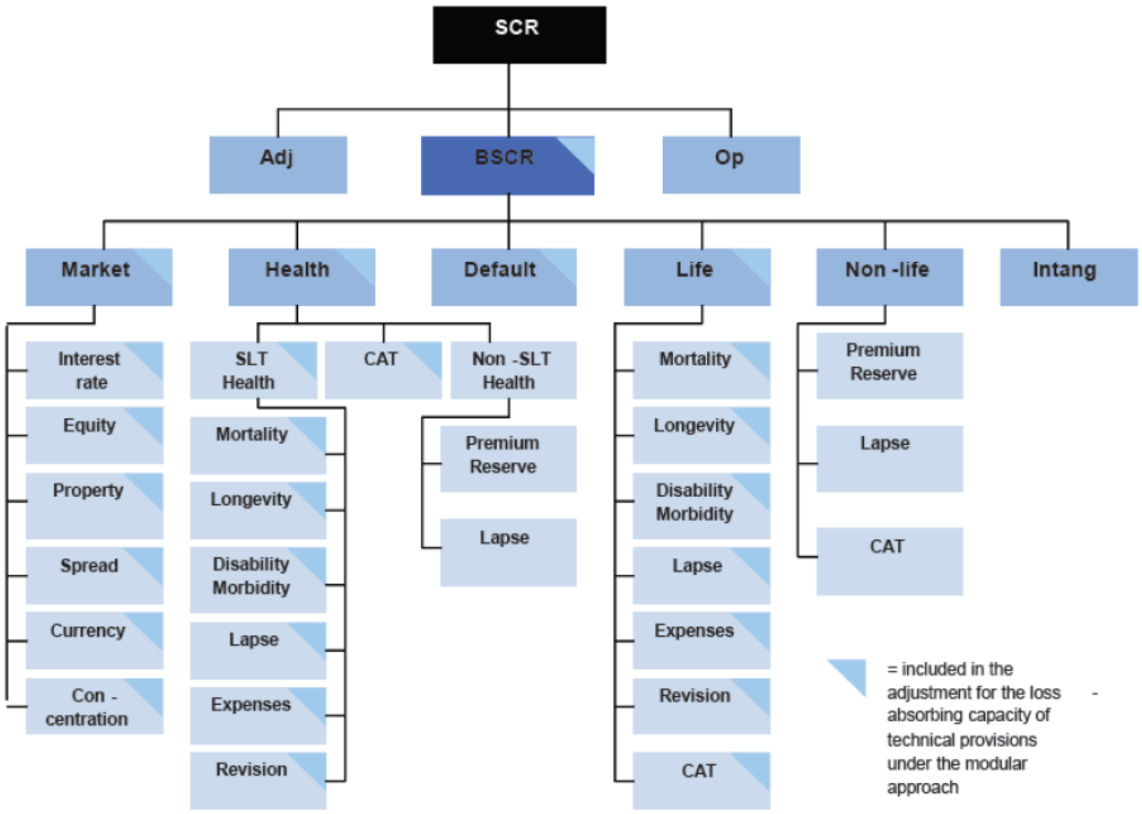


Figure 2: SCR: risk modules breakdown (Source: EIOPA)