Kriging for arbitrage-free construction of financial term-structures

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Introduction

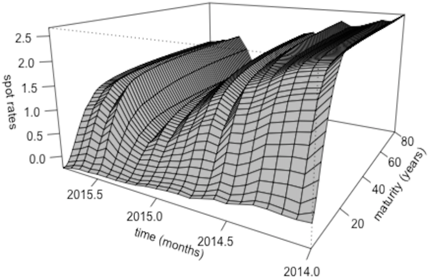
- Areski Cousin, Hassan Maatouk, Didier Rullière
  Kriging of financial term-structure, EJOR, 2016
- Areski Cousin, Djibril Gueye
  Kriging for arbitrage-free construction of volatility surfaces, working paper

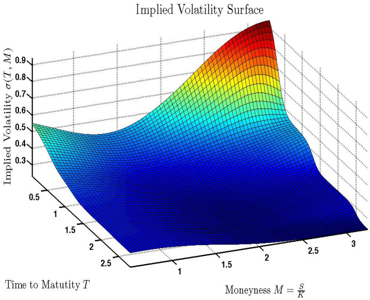
Motivation

- Learn a mapping f representing the evolution of a reference quantity Y as a function of some selected factors or explanatory variables X :

$$Y = f(X) \text{ for } X \in D \subset \mathbb{R}^d$$

- From observations of (input, output) couples : $(x_i, y_i), i = 1, \cdots, n$
- **Examples** : interest rates, default rates, implied volatilities, CVA exposures, mortality rates, surrender rates, computer experiments, any spacial data





Motivation

In risk management applications, this learning/construction problem typically has the following characteristics :

- Incomplete information : the response variable Y is only known or can only be estimated for a small set of input locations
- Indirect observation : the response variable may not be directly observed. (Typical when constructing ZC rate curves based on market quotes of some IR products)
- Noisy measurement : observed data may not be fully reliable (ex : price of illiquid instruments, Monte Carlo estimates, any statistical estimates)
- Known shape constraints : bounds on the response variable, monotonicity, convexity, ...

Motivation

What is kriging ?

- Kriging is a semi-parametric Bayesian estimation method also known as Gaussian Process Regression (or GP)
- It is a particular kernel machine learning method (see Rasmussen and Williams (2006)) but compared to frequentist machine learning techniques (support vector machines, neural network), GP estimates uncertainty
- Kriging also extends spline interpolation to uncertainty quantification : the kriging mean predictor is a spline function (curve in a RKHS with minimum norm, see Wahba (1990) or Bay el al. (2016))
- Implementation infrastructure is mature : R : DiceKriging and constrKriging, Matlab : GPML , Python : GPyTorch

Motivation

Kriging and risk management in the literature

- GP as surrogate model (estimation based on computer experiments) : Liu and Staum (2010), Ludkovski (2018), Ludkovski and Risk (2018), De Spiegeleer et al. (2018), Crépey and Dixon (2019)

- Kriging applied to real-world data (model-free) : Asgharian et al. 2013, Cousin et al. (2016), Ludkowski, Risk, Zail (2018)

- Portfolio optimization : da Barrosa et al. (2016)

- Time-series modelling : Roberts et al. (2013)

Our contributions :

- Show that kriging is a suitable tool for constructing financial term-structures and quantifying uncertainty
- Extend classical kriging to indirect observations, noisy measurements and shape-constraints

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1 The term-structure construction problem

2 Classical kriging

3 Kriging with shape constraints

The term-structure construction problem

1) Compatibility with market data :

- We observe the market quotes $\mathbf{S} = (S_1, \ldots, S_n)$ of n liquidly traded instruments
- $\mathbf{S}$ depend on the value of f at m input locations $X = (x_1, \ldots, x_m)$

The vector of output values $f(X) := (f(x_1), \ldots, f(x_m))^\top$ satisfies a linear system of the form

$$A(\mathbf{S}) \cdot f(X) = \boldsymbol{b}(\mathbf{S}),$$

where

- $A(\mathbf{S})$ is a $n \times m$ real-valued matrix
- $\boldsymbol{b}(\mathbf{S})$ is a n-dimensional column vector

$n < m \Longrightarrow$ indirect and partial information on the curve values at $x_1, \ldots, x_m$

2) No-arbitrage assumption : f is e.g. a decreasing or a convex function

The term-structure construction problem

Example 1 : OIS discount curve

Construction of function $T \to D(t_0, T)$ from $S_i, i = 1, \ldots, n$, where

- S_i : par rate at quotation date t_0 of an OIS with maturity T_i
- $t_1 < \cdots < t_{p_i} = T_i$: fixed-leg payment dates (annual time grid)
- δ_k : year fraction of period (t_{k-1}, t_k)

$$S_i \sum_{k=1}^{p_i - 1} \delta_k D(t_0, t_k) + (S_i \delta_{p_i} + 1)\, D(t_0, T_i) = 1, \quad i = 1, ..., n$$

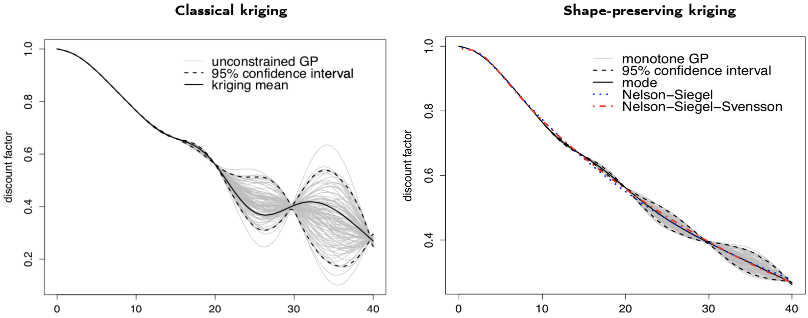
where $D(t_0, T)$ is the OIS discount factor with maturity T

The arbitrage-free curve $T \to D(t_0, T)$ is decreasing and $D(t_0, t_0) = 1$

The term-structure construction problem

Example 1 : OIS discount curve

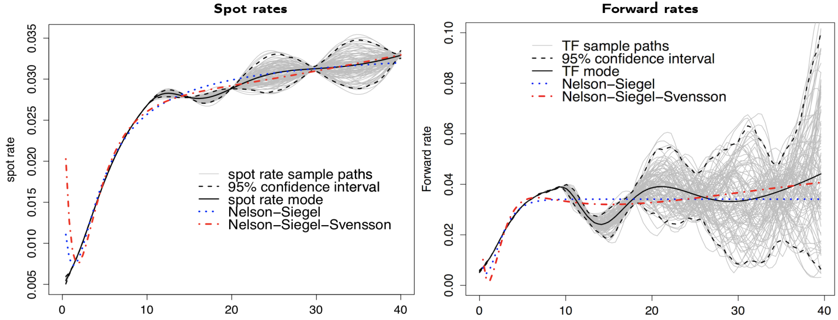
- Data : quoted swap rates as of June 3, 2010, for OIS with maturities $1y, \ldots, 10y, 15y, 20y, 30y, 40y$
- Classical kriging (left) vs kriging with monotonicity constraint (right)



The term-structure construction problem

Example 1 : OIS discount curve

- Corresponding spot rate and forward rate curves
- Monotonic kriging - GP prior with Matérn 5/2 kernel - no noise



The term-structure construction problem

Example 2 : Default rates implied from CDS spreads

- S_i : CDS spread at time t_0 with maturity T_i
- $t_1 < \cdots < t_{p_i} = T_i$: trimestrial premium payment dates, δ_k : year fraction of period (t_{k-1}, t_k)
- $D(t_0, T)$ is the discount factor associated with maturity date T
- R : expected recovery rate of the reference entity

$$S_i \sum_{k=1}^{p_i} \delta_k D(t_0, t_k) Q(t_0, t_k) = -(1 - R) \int_{t_0}^{T_i} D(t_0, u) dQ(t_0, u)$$

where $T \to Q(t_0, T)$ is the $\mathcal{F}_{t_0}$-conditional (risk-neutral) survival distribution of the reference entity, i.e.,

$$Q(t_0, T) = \mathbb{Q}(\tau > T \mid \mathcal{F}_{t_0})$$

The term-structure construction problem

Example 2 : Default rates implied from CDS spreads (cont.)

Using an integration by parts, the survival function $u \to Q(t_0, u)$ satisfies a linear relation :

$$S_i \sum_{k=1}^{p_i} \delta_k D(t_0, t_k) Q(t_0, t_k) + (1 - R) D(t_0, T_i) Q(t_0, T_i)$$
$$+ (1 - R) \int_{t_0}^{T_i} f(t_0, u) D(t_0, u) Q(t_0, u) du = 1 - R, \; i = 1, \ldots, n$$

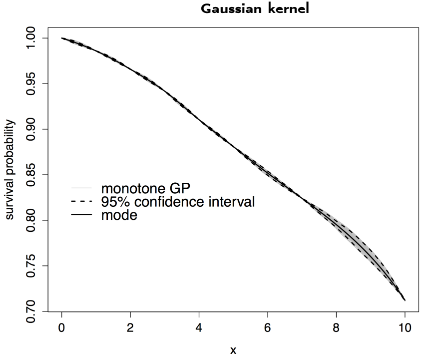
where $f(t_0, u)$ is the instantaneous forward (discount) rate associated with maturity date u.

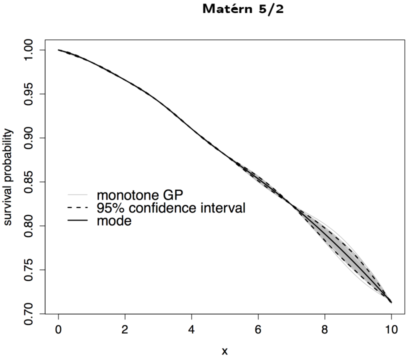
As a survival function, $T \to Q(t_0, T)$ **shall be decreasing and such that** $Q(t_0, t_0) = 1$

The term-structure construction problem

Example 2 : Default rates implied from CDS spreads (cont.)

- CDS spreads for protection maturities 1y, 2y, 3y, 4y, 5y, 7y, 10y
- Russian sovereign debt, quotes as of 06/01/2005
- Monotonic kriging - GP without noise

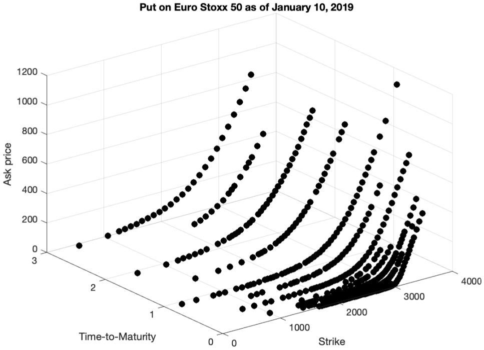




The term-structure construction problem

Example 3 : Volatility surface

We observe at t_0, a series of put option prices $f(x_i) = P((K_i, T_i))$ for different characteristics $x_i = (K_i, T_i)$, $i = 1, \ldots, n$.



The term-structure construction problem

Example 3 : Volatility surface

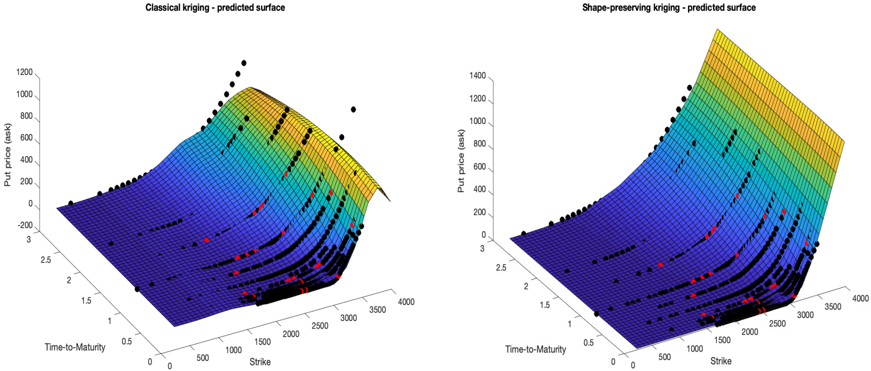
The put price surface $(K, T) \rightarrow P(K, T)$ is free of static arbitrage if

- $K \rightarrow P(K, T)$ is a convex function such that $P(0, T) = 0$ and $\frac{\partial P}{\partial K}(0, T) = 0$, for any $T \geq 0$
- $T \rightarrow P(K, T)$ is a non-decreasing function, for any $K \geq 0$
- $P(K, 0) = (K - S_0)^+$ where S_0 is the spot price.

The term-structure construction problem

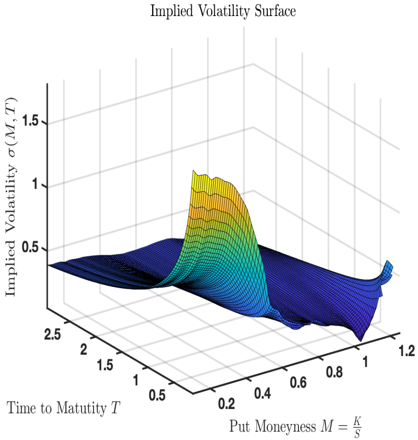
Example 3 : Volatility surface

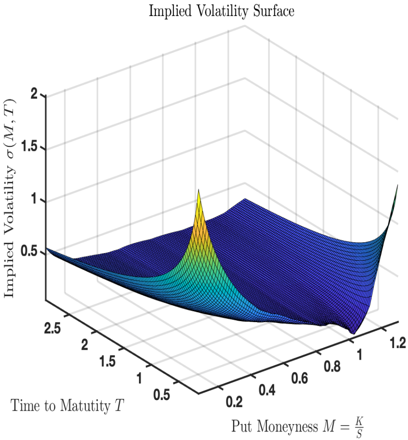
- Data : Euro Stoxx 50 Put prices as of January 10, 2019
- 5% of the data used (red points)
- Classical kriging (left) vs kriging with no-arbitrage constraints (right)



The term-structure construction problem

Example 3 : Volatility surface

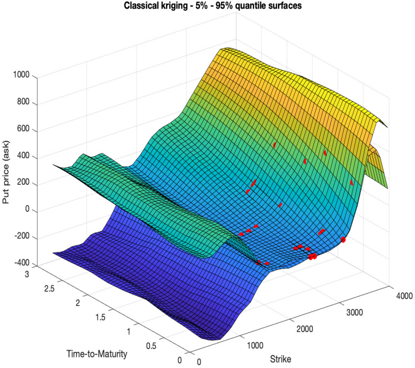


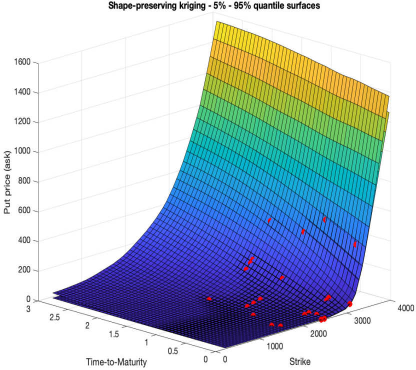


The term-structure construction problem

Example 3 : Volatility surface

- 5% and 95% estimated quantiles of the fitted GP
- Classical kriging (left) vs kriging with no-arbitrage constraints (right)





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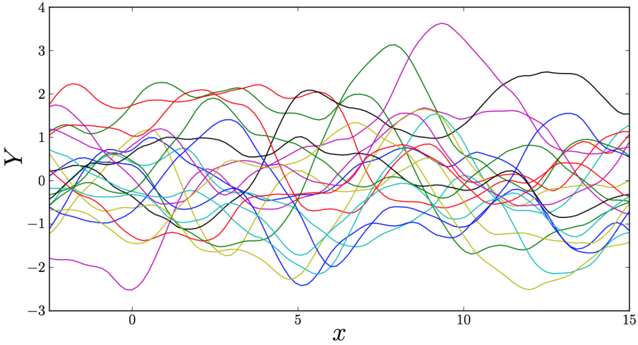
Classical kriging

Estimation of the unknown function f using Bayesian statistics

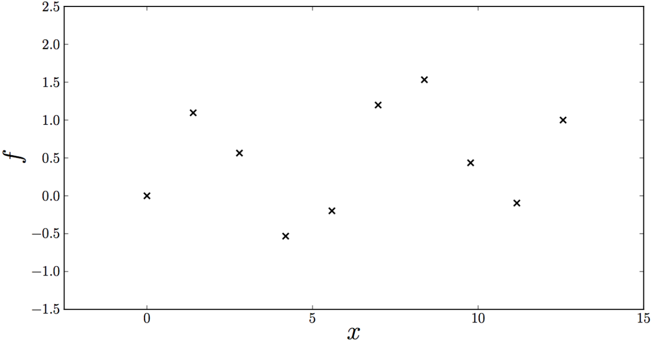
Our first belief in f is given as a Gaussian process prior Y



Classical kriging

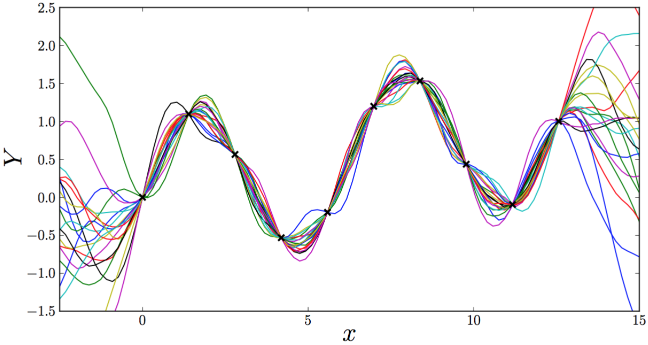
The function f is known at some input points $x^1, \dots, x^n$:

$$f\left(x^1\right) = y^1, \dots, f\left(x^n\right) = y^n.$$



Classical kriging

This belief is updated given that $Y(x_1) = y_1, \ldots, Y(x_n) = y_n$



Source : presentation of N. Durrande

Classical kriging

Definition : Gaussian process (GP) or Gaussian random field

A Gaussian process is a collection of random variables, any finite number of which have (consistent) joint Gaussian distributions.

A Gaussian process $\left(Y(x), x \in \mathbb{R}^d\right)$ is characterized by its mean function

$$\mu : x \in \mathbb{R}^d \longrightarrow \mathbb{E}(Y(x)) \in \mathbb{R}.$$

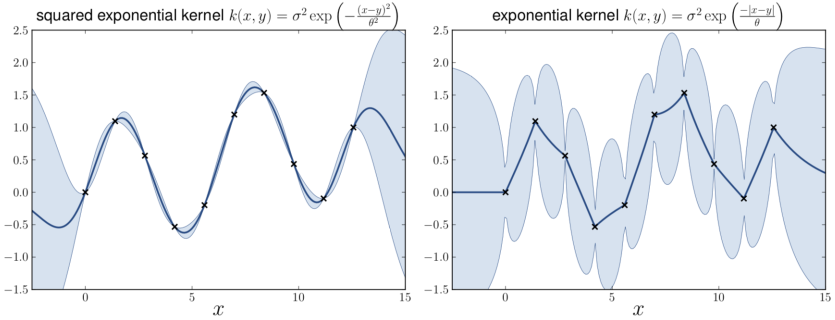
and its covariance function

$$K : (x, x') \in \mathbb{R}^d \times \mathbb{R}^d \longrightarrow \mathrm{Cov}(Y(x), Y(x')) \in \mathbb{R}.$$

1D kriging kernel	$K(x, x')$	Class
Gaussian	$\sigma^2 \exp\left(-\frac{(x-x')^2}{2\theta^2}\right)$	$\mathcal{C}^\infty$
Matérn 5/2	$\sigma^2 \left(1 + \frac{\sqrt{5}\lvert x-x'\rvert}{\theta} + \frac{5(x-x')^2}{3\theta^2}\right) \exp\left(-\frac{\sqrt{5}\lvert x-x'\rvert}{\theta}\right)$	$\mathcal{C}^2$
Matérn 3/2	$\sigma^2 \left(1 + \frac{\sqrt{3}\lvert x-x'\rvert}{\theta}\right) \exp\left(-\frac{\sqrt{3}\lvert x-x'\rvert}{\theta}\right)$	$\mathcal{C}^1$
Exponential	$\sigma^2 \exp\left(-\frac{\lvert x-x'\rvert}{\theta}\right)$	$\mathcal{C}^0$

Classical kriging

Changing the kernel K has a huge impact on the model



Source : presentation of N. Durrande

Classical kriging - indirect observations with noise

Assume that f is known up to solving a linear equality system with measurement errors :

$$A \cdot f(X) + \varepsilon = \boldsymbol{b}. \tag{1}$$

where

- A is a given $n \times m$ matrix
- $X = (x_1, \ldots, x_m)^\top \in \mathbb{R}^{m \times d}$
- $f(X) = (f(x_1), \ldots, f(x_m))^\top \in \mathbb{R}^m$
- $\boldsymbol{b} \in \mathbb{R}^n$
- ε is zero-mean Gaussian noise in $\mathbb{R}^n$ with covariance matrix Σ_{noise}
- ε is assumed to be independent of the GP Y

Classical kriging - indirect observations with noise

- $X = (x_1, \ldots, x_m)^\top \in \mathbb{R}^{m \times d}$: some design points
- $\boldsymbol{b} = (b_1, \ldots, b_n)^\top \in \mathbb{R}^n$: right-hand side of the linear system
- $Y(X) = (Y(x_1), \ldots, Y(x_m))$: vector composed of Y at point $\boldsymbol{X}$

The conditional process is still a Gaussian Process

Let Y be a GP with mean μ and covariance function K. The conditional process $Y \mid AY(X) + \varepsilon = \boldsymbol{b}$ is a GP with mean function

$$\eta(x) = \mu(x) + (A\boldsymbol{k}(x))^\top \left(A\mathbb{K}A^\top + \Sigma_{noise}\right)^{-1} (\boldsymbol{b} - A\boldsymbol{\mu}), \quad x \in \mathbb{R}^d$$

and covariance function $\tilde{K}$ given by

$$\tilde{K}(x, x') = K(x, x') - (A\boldsymbol{k}(x))^\top \left(A\mathbb{K}A^\top + \Sigma_{noise}\right)^{-1} A\boldsymbol{k}(x'), \quad x, x' \in \mathbb{R}^d$$

where $\boldsymbol{\mu} = \mu(X) = (\mu(x_1), \ldots, \mu(x_m))^\top$, $\mathbb{K}$ is the covariance matrix of $Y(X)$, $\boldsymbol{k}(x) = (K(x, x_1), \ldots, K(x, x_m))^\top$

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Shape-preserving kriging

New formulation of the problem : estimation of an unknown real-valued function $f : [0,1]^d \to \mathbb{R}$ given that

$$\begin{cases} A \cdot f(X) + \varepsilon = \boldsymbol{b} \\ f \in \mathcal{M} \end{cases}$$

where $\mathcal{M}$ is a convex set of functions satisfying some shape property.

For instance, $\mathcal{M}$ can be :

- $\mathcal{M}_0^d := \{f \in \mathcal{C}([0,1]^d, \mathbb{R}) \mid y_{\text{min}} \leq f(x) \leq y_{\text{max}},\ \forall x \in D\}$
- $\mathcal{M}_1^1 := \{f \in \mathcal{C}([0,1], \mathbb{R}) \mid f \text{ is non-decreasing}\}$
- $\mathcal{M}_2^1 = \{f \in \mathcal{C}([0,1], \mathbb{R}) \mid f \text{ is convex}\}$
- $\mathcal{M}_{12}^2 = \{f \in \mathcal{C}([0,1]^2, \mathbb{R}) \mid f \text{ is non-decreasing in } x \text{ and convex in } y\}$

Shape-preserving kriging

Main issues :

- The posterior process is not Gaussian anymore.
- The shape condition is usually infinite-dimensional.

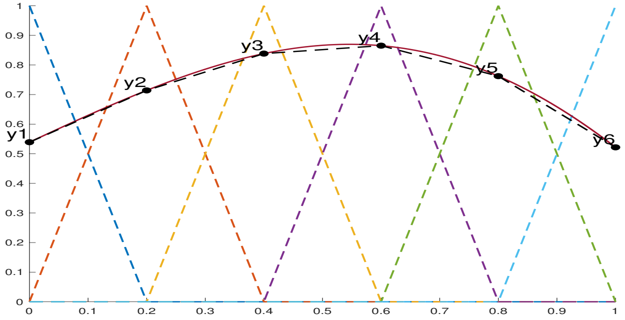
Proposed solutions :

- We construct a finite-dimensional approximation of Y for which the shape condition is easy to check.
- We consider the mode of the posterior distribution (as opposed to the posterior mean) as a new response surface estimator
- Hyper-parameters are estimated using MLE

Finite-dimensional approximation of GP (1d case)

As in Maatouk and Bay (2014), Cousin et al. (2016), López et al. (2018), we rely on basis function approximation.

- Input domain D is discretized on a regular subdivision $u_0 < \ldots < u_N$ with a constant mesh δ.
- For each u_i, we consider hat functions $\phi_i(x) := \max\left(1 - \frac{|x-u_i|}{\delta}, 0\right)$
- Y is approximated on D by $Y^N(x) = \sum_{i=0}^N Y(u_i)\phi_i(x)$



Finite-dimensional approximation of GP (1d case)

Proposition

Let Y be a zero-mean GP with covariance function K and almost surely continuous paths.

- The finite-dimensional process $Y^N(\cdot) = \sum_{i=0}^N Y(u_i)\phi_i(\cdot)$ uniformly converges to Y on D as $N \to \infty$, almost surely.
- $Y^N(x) = \mathbf{\Phi}(x)\boldsymbol{\xi}$ where $\boldsymbol{\xi} := (Y(u_0), \ldots, Y(u_N))^\top$ is a zero-mean Gaussian vector with covariance matrix Γ^N such that $\Gamma^N_{i,j} = K(u_i, u_j)$

Shape-preserving conditions :

- Y^N takes values on $[y_{\min}, y_{\max}]$ if and only if $y_{\min} \leq \xi_i \leq y_{\max}$
- Y^N is non-decreasing on D if and only if $\xi_{i+1} \geq \xi_i$
- Y^N is convex on D if and only if $\xi_{i+2} - \xi_{i+1} \geq \xi_{i+1} - \xi_i$
- $\ldots$

Finite-dimensional approximation of GP (2d case)

- $D = [0,1]^2$ is discretized on a $(N_x + 1) \times (N_t + 1)$ regular grid with knots (u_i, v_j), $i = 1, \ldots, N_x$, $j = 1, \ldots, N_t$.
- For each knot (u_i, v_j), we consider tensor product basis functions

$$\phi_{i,j}(x,t) := \max\left(1 - \frac{|x - u_i|}{\delta_x}, 0\right) \max\left(1 - \frac{|t - v_j|}{\delta_t}, 0\right)$$

- Y is approximated on D by

$$Y^N(x,t) = \sum_{i=0}^{N_x} \sum_{j=0}^{N_t} Y(u_i, v_j) \phi_{i,j}(x,t)$$

- $N = (N_x + 1)(N_t + 1)$ is the number of knots

Finite-dimensional approximation of GP (2d case)

Proposition

Let Y be a zero-mean GP with covariance function K and with almost surely continuous paths.

- The finite-dimensional process Y^N uniformly converges to Y on D as $N_x \to \infty$ and $N_t \to \infty$, almost surely.
- $Y^N(x) = \mathbf{\Phi}(x)\boldsymbol{\xi}$ where $\boldsymbol{\xi} := (Y(u_0, v_0), Y(u_0, v_1), \ldots, Y(u_{N_x}, v_{N_t}))^\top$ is a zero-mean Gaussian vector with $N \times N$ covariance matrix Γ^N such that $\Gamma^N = K((u_{i_1}, v_{j_1}), (u_{i_2}, v_{j_2}))$.

Shape-preserving conditions :

- Y^N is bounded on $[y_{\min}, y_{\max}]$ if and only if $y_{\min} \leq \xi_{i,j} \leq y_{\max}$
- $Y^N(x,t)$ is non-decreasing function of x if and only if $\xi_{i+1,j} \geq \xi_{i,j}$
- $Y^N(x,t)$ is a convex function of x if and only if $\xi_{i+2,j} - \xi_{i+1,j} \geq \xi_{i+1,j} - \xi_{i,j}$
- $\ldots$

Kriging under shape-preserving conditions

Consider a zero-mean GP prior Y with covariance function K and N-dimensional approximation Y^N.

Kriging the unknown function f boils down to finding the conditional distribution of Y^N given

$$\begin{cases} A \cdot Y^N(X) + \varepsilon = \boldsymbol{b} \\ Y^N \in \mathcal{M} \end{cases}$$

This is equivalent to finding the distribution of the truncated Gaussian vector $\boldsymbol{\xi} \sim \mathcal{N}(0, \Gamma^N)$ given that

$$\begin{cases} A \cdot \boldsymbol{\Phi}(X) \cdot \boldsymbol{\xi} + \varepsilon = \boldsymbol{b} \\ \boldsymbol{\xi} \in \mathcal{C}_{ineq} \end{cases}$$

where $\mathcal{C}_{ineq}$ is a set of linear inequality constraints.

Estimation of hyper-parameters

- We consider d-dimensional anisotropic stationary kernels :

$$K(x, x') = \sigma^2 \prod_{i=1}^{d} K_i(x_i - x_i'; \theta_i)$$

  where K_i is stationary kernel : Gaussian, Matérn 5/2, Matérn 3/2, Exponential.

- Homoscedastic noise : $\Sigma_{noise} = \sigma_{noise} \mathbb{I}_n$
- **Hyper-parameters** : $\boldsymbol{p} = (\sigma, \theta_1, \ldots, \theta_d, \sigma_{noise})$

Estimation of hyper-parameters

Following López-Lopera et al (2017), we consider two MLE approaches

- **Unconditional likelihood** : Find $\boldsymbol{p}$ that maximizes the Gaussian likelihood $\mathbb{P}\left(A \cdot \boldsymbol{\Phi}(X) \cdot \boldsymbol{\xi} + \boldsymbol{\varepsilon} = \boldsymbol{b} \mid \boldsymbol{p}\right)$ or log-likelihood

$$\mathcal{L}_N(\boldsymbol{p}) := -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log|C| - \frac{1}{2}\boldsymbol{b}^\top C^{-1}\boldsymbol{b}$$

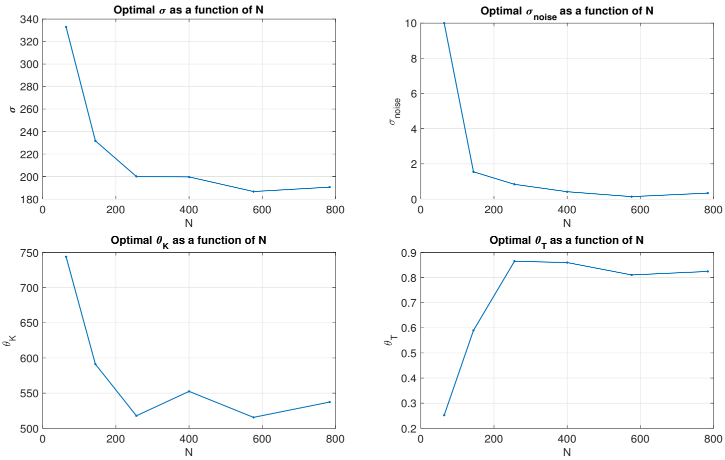
where $C := A\boldsymbol{\Phi}(X)\Gamma^N(\boldsymbol{p})\boldsymbol{\Phi}(X)A^\top + \Sigma_{noise}(\boldsymbol{p})$

- **Conditional likelihood** : Find $\boldsymbol{p}$ that maximizes the conditional probability $\mathbb{P}\left(A \cdot \boldsymbol{\Phi}(X) \cdot \boldsymbol{\xi} + \boldsymbol{\varepsilon} = \boldsymbol{b} \mid \boldsymbol{\xi} \in \mathcal{C}_{ineq}, \boldsymbol{p}\right)$ or the log-likelihood

$$\mathcal{L}_{N,cond}(\boldsymbol{p}) := \mathcal{L}_N(\boldsymbol{p}) + \log\mathbb{P}(\boldsymbol{\xi} \in \mathcal{C}_{ineq} \mid A\cdot\boldsymbol{\Phi}(X)\cdot\boldsymbol{\xi} + \boldsymbol{\varepsilon} = \boldsymbol{b}) - \log\mathbb{P}(\boldsymbol{\xi} \in \mathcal{C}_{ineq})$$

Estimation of hyper-parameters

Convergence of optimal parameter as a function of N (number of basis functions)



Mode estimator

We define the (a posteriori) most probable response surface and measurement noises as

$$\begin{cases} M_K^N(x) := \boldsymbol{\Phi}(x) \cdot (c_1^*, \ldots, c_N^*)^\top, \; x \in D \\ \boldsymbol{e}^* := (e_1^*, \ldots, e_n^*)^\top \end{cases}$$

where $(\boldsymbol{c}^*, \boldsymbol{e}^*)$ is the mode of the truncated Gaussian vector $(\boldsymbol{\xi}, \boldsymbol{\varepsilon})$ given the constraints, defined as solution of

$$\max_{\boldsymbol{c}, \boldsymbol{e}} \mathbb{P} \left(\boldsymbol{\xi} \in [\boldsymbol{c}, \boldsymbol{c} + d\boldsymbol{c}], \boldsymbol{\varepsilon} \in [\boldsymbol{e}, \boldsymbol{e} + d\boldsymbol{e}] \mid A \cdot \boldsymbol{\Phi}(X) \cdot \boldsymbol{\xi} + \boldsymbol{\varepsilon} = \boldsymbol{b}, \; \boldsymbol{\xi} \in \mathcal{C}_{ineq} \right).$$

The mode $(\boldsymbol{c}^*, \boldsymbol{e}^*)$ is solution of a quadratic problem

$$\min_{A \cdot \boldsymbol{\Phi}(X) \cdot \boldsymbol{c} + \boldsymbol{e} = \boldsymbol{b}, \; \boldsymbol{c} \in \mathcal{C}_{ineq}} \left(\boldsymbol{c}^\top (\Gamma^N)^{-1} \boldsymbol{c} + \boldsymbol{e}^\top \Sigma_{noise}^{-1} \boldsymbol{e} \right)$$

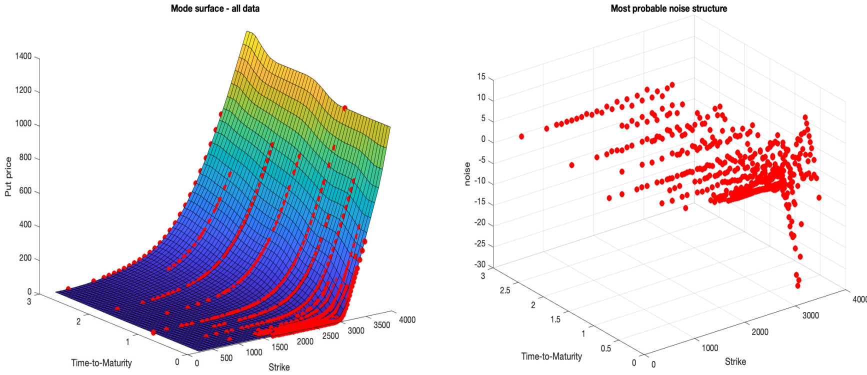
Mode estimator

The mode estimator has several advantages (over alternative estimators) :

- It satisfies the constraints on the entire domain D
- It is easy to compute as the solution of a quadratic optimisation problem
- It corresponds to the maximum a posteriori estimator in the sense of Bayesian statistics
- As N tends to infinity, the limit of M_K^N corresponds to a constrained spline that depends on K (Bay et al., 2016)

Mode estimator

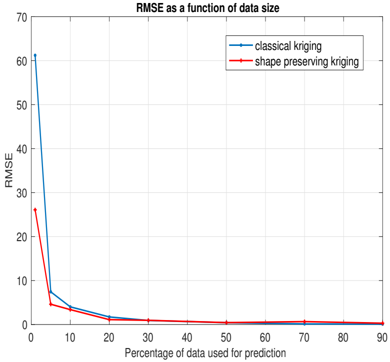
- Data : Euro Stoxx 50 Put prices as of January 10, 2019
- Fitted Gaussian kernel using uncond. MLE, all data used
- Most probable surface (left) vs most probable noise values (right)



Mode estimator - prediction accuracy

RMSE as a function of data size

- We construct a series of data subsets with increasing number of points
- We apply classical kriging and shape-preserving kriging on these subsets
- For each data size, we compute average RMSE wrt the original data set.



Sampling finite-dimensional GP with shape constraints

First remark that the distribution of $\boldsymbol{\xi}$ given $A \cdot \boldsymbol{\Phi}(X) \cdot \boldsymbol{\xi} + \varepsilon = \boldsymbol{b}$ is multinormal $\mathcal{N}(\boldsymbol{\mu}_{cond}, \Sigma_{cond})$ where

$$\begin{cases} \boldsymbol{\mu}_{cond} = \Gamma^N B^\top \left(B \Gamma^N B^\top + \Sigma_{noise} \right)^{-1} \boldsymbol{b} \\ \Sigma_{cond} = \Gamma^N - \Gamma^N B^\top \left(B \Gamma^N B^\top + \Sigma_{noise} \right)^{-1} B \Gamma^N \end{cases}$$

with $B = A \cdot \boldsymbol{\Phi}(X)$.

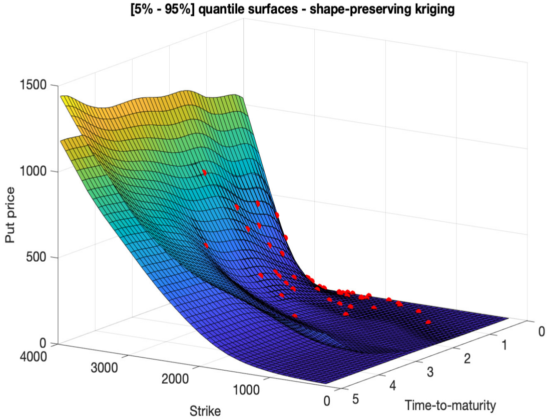
Following López-Lopera et al (2017), we consider the Hamiltonian Monte Carlo method introduced by Pakman and Paninski (2013) for sampling truncated multivariate Gaussians :

$$\mathcal{TN}\left(\boldsymbol{\mu}_{cond}, \Sigma_{cond}, \mathcal{C}_{ineq}\right)$$

MCMC initialized using the mode estimator since it satisfies the inequality constraints .

Sampling finite-dimensional GP with shape constraints

- We extrapolate the GP in T direction (adding 2 years)
- 5% and 95% estimated pointwise quantiles



Thanks for your attention.

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